## DEPARTMENT OF CIVIL ENGINEERING, IIT MADRAS

## CE 3310: ADVANCED STRUCTURAL ANALYSIS

Class: 3/4 B Tech (Civil)
Maximum Marks: 50

END SEMESTER EXAM
' $F$ ' Slot

November 28, 2011
Time Limit: 180 minutes

## Answer Question 1 (10 marks) and any FIVE other questions (8 marks each).

1. Answer any four of the following ( $21 / 2$ marks each):
(a) Distribute the applied moment in the frame shown in Fig. 1 appropriately to the various frame elements and hence draw the bending moment diagram. Also sketch the deflected shape.

(b) Consider the axially loaded system in Fig. 2. Derive an expression for the deflection at B, in terms of $P, a$ and $E A$ and draw the axial force diagram.
(c) Fig. 3 depicts the global coordinates in a non-prismatic fixed beam ABC (fixed at ends A and C). Using the conventional stiffness approach, generate the $4 \times 4$ transformation matrices, $\mathbf{T}^{\mathbf{1}}$ and $\mathbf{T}^{\mathbf{2}}$, for the two elements, marking the linking global coordinates, and hence, the fixed end force vector, $\mathbf{F}_{\mathbf{f}}=\left[\begin{array}{l}\mathbf{F}_{\mathbf{f A}} \\ \mathbf{F}_{\mathbf{f R}}\end{array}\right]$, due to a uniformly distributed load of $12 \mathrm{kN} / \mathrm{m}$ on the beam.
(d) Consider an element $A B$ in a plane truss, connecting the joint $A(0,1)$ to joint $B(3,5)$, where the global axes $(x, y)$ dimensions are expressed in metres. Find the axial deformation in the bar, given that A is connected to a rigid support and B deflects by 0.5 mm and -1.5 mm in the $x$ and $y$ directions respectively. Assume an element axial rigidity of 50000 kN .
(e) Generate the reduced element stiffness matrix $\tilde{\mathbf{k}}_{*}$ of a typical prismatic space frame element with 6 degrees of freedom, with local coordinates and properties, as depicted in Fig. 4 [No need for any derivation]. Also, generate the element fixed end force vector $\mathbf{F}_{* \mathbf{f}}$ given that the element is subject to a uniformly distributed load of $q_{\mathrm{o}}$ per unit length.
2. Analyse the laterally loaded single bay two-storeyed frame shown in Fig. 5 by an approximate force method (assuming the beams to be infinitely rigid) and draw the bending moment diagram. Hence, applying the unit load method, find an approximate value of the horizontal drift in the frame at the top, assuming flexural rigidity $E I=10000 \mathrm{kNm}^{2}$.
3. Analyse the laterally loaded single bay two-storeyed frame in Fig. 5 by the slope-deflection or moment distribution method, taking advantage of anti-symmetry in the response. Draw the bending moment diagram. Also sketch the deflection profile.


Fig. 5


Fig. 6
4. (a) Generate the force transformation $\left(\mathbf{T}_{\mathbf{F}}\right)$ matrix (of size $5 \times 2$ ), and hence find the bar forces in the five-bar truss, loaded as shown in Fig. 6.
(b) In an unloaded condition, it is given that the five bars in Fig. 6 undergo thermal elongations given by $\mathbf{e}_{\mathbf{0}}=[2,2,3,1,1]^{\mathrm{T}} \mathrm{mm}$. Find the deflections $D_{1}$ and $D_{2}$ at A.
5. Analyse the spring-supported beam shown in Fig. 7 by any method of your choice (stiffness or flexibility) and hence (i) find the deflection at B and (ii) draw the bending moment diagram of the beam. The spring stiffness is given by $6 E I L^{3}$, where $E I$ is the flexural rigidity of the beam.

6. Analyse the two-span continuous beam shown in Fig. 8 by any matrix method of your choice (stiffness or flexibility) and draw the bending moment diagram. Assume $E I=27000 \mathrm{kNm}^{2}$.
7. Consider the symmetric grid system (plan view) shown in Fig. 9, with all the elements subjected to uniformly distributed gravity loading of $12 \mathrm{kN} / \mathrm{m}$ and having uniform flexural rigidity $E I=27000 \mathrm{kNm}^{2}$ and torsional rigidity $G J=0.2 E I$. Taking advantage of symmetry and adopting any method of your choice, find the deflection at the centre $E$ and draw the bending moment diagram and probable deflected shape of a typical element AE.

8. Analyse the portal frame shown in Fig. 10 (with an internal hinge at C, loading and support settlement of 10 mm at D , as shown) by the reduced stiffness method, ignoring axial deformations, to find (a) the horizontal deflection at B , and (b) the hogging moment at B . Assume $E I=24000 \mathrm{kNm}^{2}$ for all members.

